Angular Momentum

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The theory of angular momentum L starts from the commutation relations:

$$[L_x, L_y] = i\hbar L_z, \quad [L_y, L_z] = i\hbar L_x, \quad [L_z, L_x] = i\hbar L_y,$$

The eigenvectors of L^2 and L_z satisfy:

$$\begin{aligned} &[L^2, L_z] = 0, \\ &L^2|l\ m\rangle = \hbar^2 l(l+1)|l\ m\rangle, \quad L_z|l\ m\rangle = \hbar m|l\ m\rangle. \end{aligned}$$

Defining the ladder operators as $L_\pm = L_x \pm iL_y$, they satisfy

$$L_\pm|l\ m\rangle = \hbar\sqrt{l(l+1) - m(m\pm 1)}|l\ (m\pm 1)\rangle.$$

Note that the value of l and m can be

$$l = 0, 1, 2, \cdots; \quad m = -l, -l+1, \cdots, l-1, l.$$

Eigenfunction of Angular Momentum

The eigenfunction of angular momentum is spherical harmonics, Y_l^m,

$$Y_l^m(\theta, \phi) = \epsilon \sqrt{\frac{(2l+1)}{4\pi} \frac{(l-|m|)!}{(l+|m|)!}} e^{im\phi} P_l^m(\cos\theta),$$

where $\epsilon = (-1)^m$ for $m \geq 0$ and $\epsilon = 1$ for $m \leq 0$ and P_l^m is the associated Legendre function.