Time Independent Perturbation Theory (Degenerate)

Byungjoon Min

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Department of Physics, Chungbuk National University

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We consider two degenerate states $|a^0\rangle$ and $|b^0\rangle$:

$$H^0|a^0\rangle = E^0|a^0\rangle,$$
$$H^0|b^0\rangle = E^0|b^0\rangle,$$

where $\langle a^0|b^0\rangle = \delta_{a,b}$. Then, in general the linear combination of a^0 and b^0 also satisfies the equation,

$$|\psi^0\rangle = \alpha|a^0\rangle + \beta|b^0\rangle,$$
$$H^0|\psi^0\rangle = E^0|\psi^0\rangle.$$

Applying the perturbation, $H = H^0 + \lambda H^1$, the degenerate values of energy are split.

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Expanding $|\psi^0\rangle$ and E^0, we have

$$\left(H^0 + \lambda H^1\right)\left(|\psi^0\rangle + \lambda|\psi^1\rangle + \lambda^2|\psi^2\rangle + \cdots\right)$$
$$= \left(E^0 + \lambda E^1 + \lambda^2 E^2 + \cdots\right)\left(|\psi^0\rangle + \lambda|\psi^1\rangle + \lambda^2|\psi^2\rangle + \cdots\right).$$

Then, the first order part will be

$$H^1|\psi^0\rangle + H^0|\psi^1\rangle = E^1|\psi^0\rangle + E^0|\psi^1\rangle.$$

First Order Correction

From the first order equation,

$$H^1|\psi^0\rangle + H^0|\psi^1\rangle = E^1|\psi^0\rangle + E^0|\psi^1\rangle,$$

applying $\langle a^0|$, we have

$$\langle a^0|H^1|\psi^0\rangle + \langle a^0|H^0|\psi^1\rangle = \langle a^0|E^1|\psi^0\rangle + \langle a^0|E^0|\psi^1\rangle.$$
$$\langle a^0|H^1|\psi^0\rangle + E^0\langle a^0|\psi^1\rangle = E^1\langle a^0|\psi^0\rangle + E^0\langle a^0|\psi^1\rangle.$$
$$\langle a^0|H^1|[\alpha|a^0\rangle + \beta|b^0\rangle] = E^1\langle a^0|[\alpha|a^0\rangle + \beta|b^0\rangle].$$

Finally, we have

$$\alpha\langle a^0|H^1|a^0\rangle + \beta\langle a^0|H^1|b^0\rangle = \alpha E^1.$$

Similarly applying $\langle b^0|$, we have

$$\alpha\langle b^0|H^1|a^0\rangle + \beta\langle b^0|H^1|b^0\rangle = \beta E^1.$$

First Order Correction

Solving these two equations, we have the first order correction E^1:

$$\alpha\langle a^0|H^1|a^0\rangle + \beta\langle a^0|H^1|b^0\rangle = \alpha E^1.$$
$$\alpha\langle b^0|H^1|a^0\rangle + \beta\langle b^0|H^1|b^0\rangle = \beta E^1.$$

Thus, we have to solve the eigenvalue equation:

$$\begin{pmatrix} \langle a^0|H^1|a^0\rangle & \langle a^0|H^1|b^0\rangle \\ \langle b^0|H^1|a^0\rangle & \langle b^0|H^1|b^0\rangle \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = E^1 \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

Check the example (7.2) in Griffiths.