Assignment 4

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(due date: October 15, 2018)

1 (Essential) Special Functions [0 pt]

Study by yourself Spherical Harmonics, Legendre function, Laguerre function, and Spherical Bessel function. See "Mathematical Methods in the Physical Sciences" by Boas or "Mathematical Methods for Physicists" by Arfken and Weber. See also

- Spherical Harmonics `https://en.wikipedia.org/wiki/Spherical_harmonics`
- Associated Legendre Polynomials `https://en.wikipedia.org/wiki/Associated_Legendre_polynomials`
- Laguerre Polynomials `https://en.wikipedia.org/wiki/Laguerre_polynomials`
- Spherical Bessel Functions `https://en.wikipedia.org/wiki/Bessel_function#Spherical_Bessel_functions`

2 Schrödinger Equation in Polar Coordinates [30 pt]

Derive general solutions of the Schrödinger equation in polar coordinates r, ϕ, with $x = r\cos\phi$ and $y = r\sin\phi$, for a potential that depends only on r. Using the Lapalcian operator in polar coordinates $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}$, derive the Schrödinger equation in polar coordinates:

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2\psi(r,\phi)}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2\psi(r,\phi)}{\partial \phi^2}\right] + V(r)\psi(r,\phi) = E\psi(r,\phi). \tag{1}$$

Try the solution $\psi(r,\psi) = R(r)\Phi(\phi)$. What is the radial equation for $R(r)$. What is the equation obeyed by $\Phi(\phi)$?

3 Three-Dimensional Harmonic Oscillator [20 pt]

Consider the three-dimensional harmonic oscillator with the potential:

$$\begin{aligned} V(r) &= \frac{1}{2}m\omega^2 r^2 \\ &= \frac{1}{2}m\omega^2(x^2 + y^2 + z^2). \end{aligned} \tag{2}$$

Show that separation of variables in Cartesian coordinates gives the energy spectrum,

$$E_n = \hbar\omega\left(n_x + n_y + n_z + \frac{3}{2}\right). \tag{3}$$