Assignment 7

Byungjoon Min, Quantum Mechanics II
(due date: November 27, 2018)

1 An-harmonic Oscillator [50 pt]

Consider for the quantum oscillator problem with the Hamiltonian,

$$\begin{aligned} H^0 &= \frac{p^2}{2m} + \frac{1}{2m}(m\omega x)^2 \\ &= -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2m}(m\omega x)^2. \end{aligned} \tag{1}$$

1.1 Ground State [30 pt]

Applying perturbation

$$H^1 = \alpha x^4,$$

show that the first-order correction to the ground-state energy is

$$E^1 = \frac{3}{4}\alpha\left(\frac{\hbar}{m\omega}\right)^2. \tag{2}$$

You do not need to obtain again the energy and wave functions of the harmonic oscillator problem. You can use the results of the quantum harmonic such as

$$\psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{(1/4)} \frac{1}{\sqrt{2^n n!}} H_n(z) e^{-z^2/2}, \tag{3}$$

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{(1/4)} e^{-z^2/2}, \tag{4}$$

where $z = \sqrt{\frac{m\omega}{\hbar}}x$.

1.2 (not easy) Excited States [20 pt]

Show that in general the first-order correction to the energy is

$$E_n^1 = \frac{3\hbar^2\alpha}{4m^2\omega^2}[1 + 2n + 2n^2]. \tag{5}$$

You may need to use the relation

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(\hat{a}_+ + \hat{a}_-), \tag{6}$$

where a_+ and a_- are respectively the raising and lowering operators. And, argue the physical reason why the perturbation result will break down for large n.

2 Degenerate Perturbation Theory [20 pt]

Griffith 6.9.