Assignment 9

Byungjoon Min, Quantum Mechanics II
(due date: December 10, 2018)

1 Commutation relations of S and L [20 pt]

Griffiths 6.16. This problem allows you to understand how to deal with $S \cdot L$ in the spin-orbit coupling.

2 (Easy! Do not give up!) Darwin term [30 pt]

According to Dirac's relativistic electron theory, there is an additional correction to the Hamiltonian of a hydrogen atom that takes the form

$$H_D = \frac{e^2\hbar^2}{8\epsilon_0 m^2 c^2}\delta(r). \tag{1}$$

Using the perturbation theory, show that the energy shift by the term usually referred to as the Darwin term is given by

$$E_D = -\frac{\alpha^2 E_n}{n}, \tag{2}$$

for an $l = 0$ state. Here α is the fine structure constant $\frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137.036}$.

3 Variational Principle [30 pt]

Consider an-harmonic oscillator with the Hamiltonian

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \lambda x^4. \tag{3}$$

Using a trial wave function

$$\psi(x, \alpha) = Ae^{-\alpha x^2/2}, \tag{4}$$

show that the best approximation of the energy of the ground state is

$$E(\alpha) = \frac{3}{8}\left(\frac{6\hbar^4\lambda}{m^2}\right)^{1/3}, \tag{5}$$

and the best approximation of the ground state corresponds to

$$\alpha = \left(\frac{6m\lambda}{\hbar^2}\right)^{1/3}. \tag{6}$$