Review of Quantum Mechanics I (part 1)

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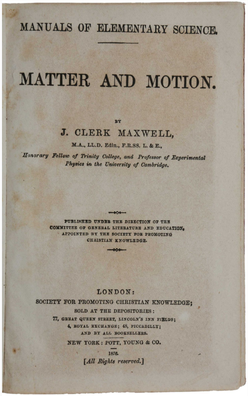
Equation of Motion



MANUALS OF ELEMENTARY SCIENCE.

MATTER AND MOTION.

BY

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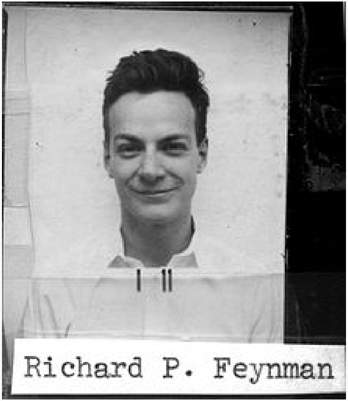

All things are made of atoms - little particles that move around in perpetual motion.

The Schrödinger equation

Schrödinger equation (Ψ)

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi.$$

Newtonian Mechanics (x, p)

$$F = -\frac{\partial V}{\partial x} = m\frac{d^2x}{dt^2}.$$

Born's Interpretation of Wave Functions

Born's Statistical Interpretation

$$\int_a^b |\Psi(x,t)|^2 dx$$

Probability of finding the particle between a and b, at time t

Normalization

If Ψ is a solution, $C\Psi$ is also a solution. We can determine C by using the normalization condition with the interpretation,

Normalization

$$\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1.$$

Expectation Values

Expectation Values

$$
\begin{aligned}
\langle x \rangle &= \int \Psi^* \hat{x} \Psi dx, \\
\langle p \rangle &= \int \Psi^* \hat{p} \Psi dx, \\
&= \int \Psi^* \left(-i\hbar \frac{\partial}{\partial x} \right) \Psi dx
\end{aligned}
$$

- Show that $\hat{p} = -i\hbar\frac{\partial}{\partial x}$ ($\odot$ assignment)

Time-Independent Schrödinger Equation

Separation of Variables

When physicists meet partial differential equations, we first try separation of variables. We assume that V is independent to t, and thus $\Psi(x,t) = \psi(x)\varphi(t)$. Substituting $\Psi(x,t)$ into the Schrödinger equation, we obtain two ordinary differential equations:

$$i\hbar \frac{d\varphi}{dt} = E\varphi, \tag{1}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi. \tag{2}$$

We call Eq. (2) as the time-independent Schrödinger equation.

Time Evolution of Wave Functions

Solving the equation

$$i\hbar\frac{d\varphi}{dt} = E\varphi,$$

we find

Time evolution

$$\varphi(t) = e^{-iEt/\hbar}.$$

Time-Independent Schrödinger Equation

Time-Independent Schrödinger Equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi.$$

Assuming that we somehow solve the time-independent Schrödinger equation, we have $\psi(x)$. In general, the equation yields an infinite collection of solutions $[\psi_1(x), \psi_2(x), \ldots]$, with its associated value of the constant $[E_1, E_2, \ldots]$.

General Solutions

All together, we have general solutions with an initial condition

$$\Psi(x,0) = \sum_{n=1}^{\infty} c_n \psi_n(x),$$

General Linear Combination of the solutions

$$\Psi(x,t) = \sum_{n=1}^{\infty} c_n \psi_n(x) e^{-iE_n t/\hbar}. \tag{3}$$

Where to go next...

Let us go to simple examples such as

1. Infinite Potential Well
2. Harmonic Osciallator
3. Free Particle