Bose-Einstein Condensation

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Bose-Einstein Condensation

Check the total number of bosons with $\mu = 0$,

$$N = \int_0^\infty g(\epsilon) \frac{1}{e^{\beta\epsilon} - 1} d\epsilon$$
$$= \frac{V m^{3/2}}{\sqrt{2}\pi^2 \hbar^3} \int_0^\infty \frac{\sqrt{\epsilon}}{e^{\beta\epsilon} - 1} d\epsilon$$
$$= V \left(\frac{\sqrt{2\pi m k_B T}}{h} \right)^3 \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{\sqrt{z}}{e^z - 1} dz.$$

where $z = \beta\epsilon$ and $\lambda = \frac{h}{\sqrt{2\pi m k_B T}}$ is the thermal de Broglie wavelength. By using the Riemann zeta function as

$$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty \frac{z^{s-1}}{e^z - 1} dz,$$

we obtain

$$N = \left(\frac{V}{\lambda^3} \right) \zeta(3/2).$$

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What does it mean?

$$\frac{N_{max}}{V} = \frac{\zeta(3/2)}{\lambda^3} \approx \frac{2.612}{\lambda^3}$$

- Should there be the maximum number of bosons? No.
- Our approximation of the distribution of eigenstates as a continuum breaks down if we try to put more particles in. In other words, all the particles more than N_{max} will be in the ground state.
- This phenomenon is called Bose-Einstein Condensation, first predicted in 1924 and experimentally confirmed in 1995.

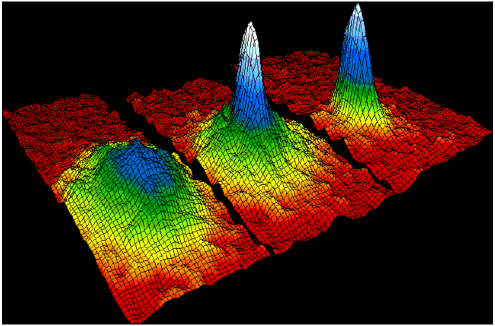
Bose-Einstein Condensation

When and why does it happen?

$$k_B T_c^{(BEC)} = \frac{h^2}{2\pi m} \left(\frac{N}{V \zeta(3/2)} \right)^{3/2}.$$

When the distance between μ and the lowest energy level ϵ_0 is significantly smaller than the distance between the lowest and the second lowest energy level ϵ_1, the continuum approximation becomes qualitatively wrong. Then, the lowest state (ground state) absorbs all the extra particles beyond N_{max}.

Bose-Einstein Condensation



Velocity-distribution for a gas of rubidium atoms, confirming the Bose-Einstein Condensation [from Wikipedia].