Classical Statistical Mechanics

Byungjoon Min

Department of Physics, Chungbuk National University

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Thermodynamics and Statistical Mechanics

- Thermodynamics: Macroscopic state from macroscopic variables
- Statistical Mechanics: Macroscopic state from microscopic variables
- Classical and Quantum Statistical Mechanics

Thermodynamic Systems

- Isolated system (E,N,V) $\rightarrow$ microcanonical ensemble
- Closed system (T,N,V) $\rightarrow$ canonical ensemble
- Open system (T,μ,V) $\rightarrow$ grandcanonical ensemble

Number of State & Ensemble

Counting the number of state $\Omega \sim \Delta x \Delta p$ in the phase space is essential.

Note that we consider a large collections or Ensemble of the system, not a single system.

Fundamental Postulate in Statistical Mechanics

$$P = \frac{1}{\Omega_O}, \tag{1}$$

where Ω_O is the total number of possible states. Then, the probability of i state is

$$p_i = \frac{\Omega_i}{\Omega_O}. \tag{2}$$

And, the expectation value of an observable is

$$\begin{aligned} \langle X \rangle &= \sum_i X_i p_i \\ &= \frac{\sum_i X_i \Omega_i}{\Omega_O}. \end{aligned} \tag{3}$$

Liouville-Gibbs Theorem & Ergodicity

Foundation of the classical statistical mechanics. Liouville-Gibbs

Theorem:

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \{\rho, H\} = 0.$$

Ergodicity:

$$\langle f \rangle = \frac{\int f(q,p)\rho(q,p)dqdp}{\int \rho(q,p)dqdp}.$$

$$\bar{f} = \frac{1}{T}\int_0^T f(q,p)dt.$$

Density of States

The number of states between the energy E and $E + \Delta E$ is given by

$$\Omega(E, E + \Delta E) = g(E)\Delta E, \tag{4}$$

where $g(E)$ is the density of states.

Thus, the number (or equivalently density) of state is all that we need to know in microcanonical ensemble.

Where to go next. . .

A mathematical skill that we need to know in this course, Lagrange multiplier.