Entropy

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Thermodynamics

A branch of physics concerns with heat and temperature and their relation to energy and work.

- Zeroth law: Thermal equilibrium (Transitivity)
- First law: $Q = \Delta E + W$ (Conservation of energy)
- Second law: $\Delta S \geq 0$ (Entropy)
- Third law: $S \to C$ where C is a constant when $T \to 0$.

Thermodynamic Entropy

Thermodynamic is defined as

$$\Delta S = \frac{Q}{T}, \tag{1}$$

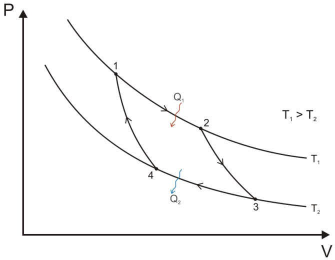
where Q is heat and T is temperature.

- Entropy never decreases, $\Delta S \geq 0$.
- Entropy is a state function (path independent).
- Arrow of time: emerging irreversibility from reversible microscopic dynamics.
- The Universe started in an unusual low entropy state (big bang), and irreversibly is moving to equilibrium (maximum entropy).

See arrow of time again with Loschmidt's paradox.

Carnot Engine

Carnot prove that the most efficient engine is a reversible one.



Carnot cycle (from Wikipedia).

The efficiency η of the Carnot engine is given by

$$\eta = \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H} = \frac{T_H - T_L}{T_H},$$

since $\frac{Q_H}{T_H} = \frac{Q_L}{T_L}$ (note that S is a state function).

Entropy as Disorder

Boltzmann Entropy is defined as

$$S = k_B \log \Omega, \tag{2}$$

where k_B is the Boltzmann constant for connecting statistical interpretation of entropy with thermodynamic entropy and Ω is the number of states.

Entropy of Mixing

Consider the configuration entropy ignoring the momentum space for $N/2$ undistinguished ideal gas white atoms on the left side and $N/2$ undistinguished ideal gas black atoms on the right side,

$$S_{unmixed} = 2k_B \log[V^{N/2}/(N/2)!].$$

Assuming white and black atoms have the same mass and energy, we can remove the partition without any irreversible change. The configuration entropy after removing the partition is

$$S_{mixed} = 2k_B \log[(2V)^{N/2}/(N/2)!].$$

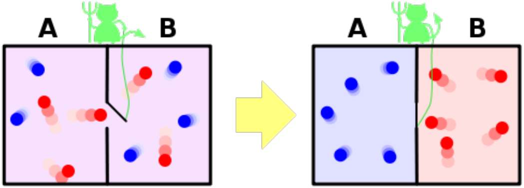
The change of entropy due to the mixing is

$$\begin{aligned}\Delta S &= S_{mixed} - S_{unmixed} \\ &= Nk_B \log 2.\end{aligned}$$

Thus, generally, we can define a counting entropy as

$$S = k_B \log \Omega.$$

Maxwell's Demon



Maxwell's Demon (from Wikipedia).

Entropy as Information

Entropy is as a measure of our ignorance (the lack of information) about a system,

$$S = -k_B \sum_i P_i \log P_i. \tag{3}$$

Let us think again "arrow of time" in an isolated system deeply. Entropy is constant for a Hamiltonian system. Entropy increases only when we ignore or exclude some degrees of freedom either external (information flow into the environment or heat bath) or internal (information flow into irrelevant microscopic degrees of freedom by coarse-graining).

Information Entropy

$$S = -\sum_i P_i \log P_i, \tag{4}$$

where the entropy is measured in “bits”.

- Entropy is maximum for equal probabilities (equilibrium).
- Entropy is unchanged by extra states of zero probability.
- Entropy changes for getting additional information (conditional probabilities).

Burning Information and Maxwell's Demon

Can we burn information as fuel for work? See Problem Sethna (5.2).

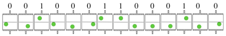


Fig. 5.10 Minimalist digital memory tape. The position of a single ideal gas atom denotes a bit. If it is in the top half of a partitioned box, the bit is one, otherwise it is zero. The side walls of the box are pistons, which can be used to set, reset, or extract energy from the stored bits. The numbers above the boxes are not a part of the tape, they just denote what bit is stored in a given position.

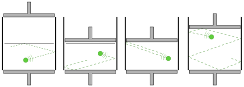


Fig. 5.11 Expanding piston. Extracting energy from a known bit is a three-step process: compress the empty half of the box, remove the partition, and retract the piston and extract $P\,dV$ work out of the ideal gas atom. (One may then restore the partition to return to an equivalent, but more ignorant, state.) In the process, one loses one bit of information (which side of the the partition is occupied).

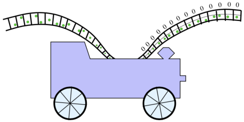


Fig. 5.12 Information-burning engine. A memory tape can therefore be used to power an engine. If the engine knows or can guess the sequence written on the tape, it can extract useful work in exchange for losing that information.

Burning information (from Sethna's book).