Metals and the Fermi Gas

Byungjoon Min

Department of Physics, Chungbuk National University

November 8, 2018

Electrons in a Metal

Consider electrons in a metal as free non-interacting electrons.

$$\langle n \rangle = \frac{1}{e^{\beta(\epsilon-\mu)} + 1},$$
$$\epsilon = \frac{\hbar^2 k^2}{2m},$$
$$g(\epsilon)d\epsilon = \frac{mV}{\pi^2 \hbar^3}\sqrt{2m\epsilon}d\epsilon.$$

Electrons in a Metal

Then, the total number of electrons is

$$N = \int_0^\infty g(\epsilon)\langle n\rangle d\epsilon,$$

$$\langle n\rangle = \frac{1}{e^{\beta(\epsilon-\mu)}+1}.$$

At $T = 0$ (equivalently $\beta \to \infty$),

- When $\epsilon \leq \mu$, $\langle n\rangle = 1$.
- When $\epsilon > \mu$, $\langle n\rangle = 0$.

Fermi Energy

We define the zero temperature value of chemical potential as Fermi energy ϵ_F. It plays an important role to analyze the (electrical) property of solids. And, we can obtain the total number of particles by using ϵ_F as

$$
\begin{aligned}
N &= \int_0^\infty g(\epsilon)\langle n\rangle d\epsilon \\
&= \int_0^{\epsilon_F} g(\epsilon) d\epsilon \\
&= \int_0^{\epsilon_F} \frac{mV}{\pi^2\hbar^3}\sqrt{2m\epsilon}d\epsilon \\
&= \frac{(2\epsilon_F m)^{3/2}}{3\pi^2\hbar^3}V.
\end{aligned}
$$

This approximation is not accurate in general because we assume independent free particles, but it is surprisingly useful as the simplest example.

Degeneracy Pressure

The Fermi energy is then

$$\epsilon_F = \frac{\hbar^2}{2m}(3\pi^2 N/V)^{2/3} \sim \left(\frac{N}{V}\right)^{2/3}.$$

The energy is

$$E = \int_0^{\epsilon_F} \epsilon g(\epsilon) d\epsilon = \int_0^{\epsilon_F} \epsilon \frac{mV}{\pi^2\hbar^3}\sqrt{2m\epsilon} d\epsilon$$
$$\sim V\epsilon_F^{5/2} = V\left(\frac{N}{V}\right)^{5/3}.$$

The degeneracy pressure is

$$P \sim \frac{E}{V} = \left(\frac{N}{V}\right)^{5/3} = \rho^{5/3}.$$