Assignment 2

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(due date: September 19, 2018)

1 Lotto [20 pt]

1.1 Winning probability

To win "Lotto" that is a lottery in Korea, one must pick 6 different numbers out of the 45 available. The order of the chosen numbers does not matter. If we buy only one ticket, what are our chances p_L of winning Lotto?

1.2 Stirling's Approximation

Compare your answer $\log p_L$ of the problem (1.1) with the simple version of Stirling's approximation ($\log n! \approx n \log n - n$). Note that $\log 45 \approx 3.807$, $\log 39 \approx 3.664$, and $\log 6 \approx 1.792$.

2 the ratio of males to females in a population [20 pt]

First assume that the probability of a baby being conceived male or female is 1/2. Next assume that in one country, people prefer sons over daughters. So, every couple endlessly tries to conceive until they have one boy. And, just after having a boy, every couple stops to try to get pregnant. Then, what will be the sex ratio if the population of the country is extremely large, $N \to \infty$. Obtain the ratio by two different approaches.

2.1 Geometric Series

Show that the difference between the number of female and male will be given by following:

$$\begin{aligned} N_f - N_m &= \left(\frac{1}{2}\right)^1 (-1) + \left(\frac{1}{2}\right)^2 (0) + \left(\frac{1}{2}\right)^3 (1) + \left(\frac{1}{2}\right)^4 (2) + \cdots \\ &= \sum_{n=0}^{\infty} \frac{1}{2^{n+1}} (n-1) \end{aligned}$$

And, calculate $N_f - N_m$ by using the geometric series.

2.2 Independent events

Justify that N events are independent each other. Then, show that the expected value of N_f and N_m will be the same as $N/2$ without any other calculations.

3 Poisson Distribution [20 pt]

The Poisson distribution is a limiting case of the binomial distribution when the probability p of success is small and the number n of trials is large.
i) Start with the binomial distribution:

$$P(x) = \binom{n}{x} p^x q^{n-x}.$$

ii) rewrite the distribution in terms of the mean $\lambda = np$, n, and x:

$$P(x) = \binom{n}{x} \left(\frac{\lambda}{n}\right)^x \left(1 - \frac{\lambda}{n}\right)^{n-x}.$$

Show that the distribution becomes the Poisson distribution when $n \to \infty$:

$$\lim_{n\to\infty} P(x) = \frac{e^{-\lambda}\lambda^x}{x!}.$$

The Poisson distribution is another important distribution in probability and statistics with many applications (`http://en.wikipedia.org/wiki/Poisson_distribution`).