Assignment 3

Byungjoon Min, Statistical Mechanics
(due date: October 3, 2018)

1 Chuseok (Hangawi) [0 pt]

Have a happy Chuseok (Hangawi) with your family!

2 Coin Flip [10 pt]

If a coin is flipped $2N$ times, show that the probability P_m that one can get $N+m$ of them heads is

$$P_m = 2^{-2N} \frac{(2N)!}{(N+m)!(N-m)!}. \tag{1}$$

3 Maxwell Relations [20 pt]

Consider the equilibrium energy $E(S, V, N)$. One knows that the second derivatives of E are symmetric; at fixed N, we get the same answer whichever order we take partial derivatives with respect to S and V. Use this to show the Maxwell relation,

$$\left(\frac{\partial T}{\partial V}\right)_{S,N} = -\left(\frac{\partial P}{\partial S}\right)_{V,N} \tag{2}$$

4 Lagrange Multipliers [20 pt]

Let

$$f(x, y) = x^2 - xy + y^2 \tag{3}$$

be the temperature distribution in the plane. Let some bug be restricted to live in a circle of radius 5 given by the constraint equation

$$g(x, y) = x^2 + y^2 - 25 = 0. \tag{4}$$

What is the hottest point in this bug's world?