Assignment 5

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(due date: October 17, 2018)

1 Burning Information and Maxwellian Demons [50 pt]

Problem (5.2) in James P. Sethna. You can see the problem and figures in his webpage `http://sethna.lassp.cornell.edu/`.

Is there a minimum energy cost for taking a measurement? In this exercise, we shall summarize the work of Szilard as presented by Bennett and Feynman. We start by addressing again the connection bettween information entropy and thermodynamic entropy. Can we burn information as fuel?

Consider a really frugal digital memory tape, with one atom used to store each bit. The tape is a series of boxes, with each box containing one ideal gas atom. The box is split into two equal pieces by a removable central partition. If the atom is in the top half of the box, the tape reads one; if it is in the bottom half the tape reads zero. The side walls are frictionless pistons that may be used to push the atom around. If we know the atom position in the nth box, we can move the other side wall in, remove the partition, and gradually retract the piston to its original position —destroying our in- formation about where the atom is, but extracting useful work.

1.1 Burning the information [10 pt]

Assuming the gas expands at a constant temperature T, how much work PdV is done by the atom as the piston retracts?

This is also the minimum work needed to set a bit whose state is currently unknown. However, a known bit can be reset for free.

[Answer. $W = k_B T \log 2$. You do not need to do any calculation.]

1.2 Rewriting a bit [10 pt]

Give a sequence of partition insertion, partition removal, and adiabatic side-wall motions that will reversibly convert a bit zero (atom on bottom) into a bit one (atom on top), with no net work done on the system. Thus the only irreversible act in using this memory tape occurs when one forgets what is written upon it (equivalent to removing and then reinserting the partition).

1.3 Forgetting a bit [10 pt]

Suppose the atom location in the nth box is initially known. What is the change in entropy, if the partition is removed and the available volume doubles? Give both the thermodynamic entropy (involving k_B) and the information entropy (involving $k_S = 1/\log 2$).

This entropy is the cost of the missed opportunity of extracting work from the bit. What prevents a Maxwellian demon from using an atom in an unknown state to extract work? The demon must first measure which side of the box the atom is on. Early workers suggested that there must be a minimum energy cost to take this measurement, equal to the energy gain extractable from the bit. Bennett argues that no energy need be expended in the measurement process. Why does this not violate the second law of thermodynamics?

1.4 Demonic states [20 pt]

After the bit has been burned, is the demon in a known state? What is its entropy? How much energy would it take to return the demon to its original state, at temperature T? Is the second law violated?

The demon can extract an unlimited amount of useful work from a tape with an unknown bit sequence if it has enough internal states to store the sequence—basically it can copy the information onto a second, internal tape. But the same work must be expended to re-zero this internal tape, preparing it to be used again. Szilard's insight might seem self-evident; one can always pay for energy from the bath with an increase in entropy.