# Assignment 6

Byungjoon Min, Statistical Mechanics
(due date: November 7, 2018)

## 1 Classical Harmonic Oscillators [30 pt]

Consider a classical harmonic oscillator with mass $m$ and frequency $w$ in one dimension. The Hamiltonian of the oscillators is given by

$$H = \frac{p_i^2}{2m} + \frac{1}{2}mw^2q_i^2. \tag{1}$$

### 1.1 Partition Function

Obtain the partition function $Z$ of a classical harmonic oscillator. That is

$$Z = \int_{-\infty}^{\infty} dqdp \frac{1}{h} e^{-\beta H},$$

### 1.2 Energy and Specific Heat

Show that

$$\langle E \rangle = k_B T, \quad C = \frac{\partial E}{\partial T} = k_B.$$

Also compare the results with that for microcanonical ensemble in assignment 4.

## 2 Quantum Harmonic Oscillators [30 pt]

Consider a quantum harmonic oscillators. The energy of the oscillators is given by

$$E_n = \hbar w \left( n + \frac{1}{2} \right).$$

### 2.1 Partition Function

Obtain the partition function $Z$ of a quantum harmonic oscillator. That is

$$Z = \sum_{n=0}^{\infty} e^{-\beta E_n},$$

### 2.2 Energy and Specific Heat

Show that

$$\langle E \rangle = \hbar w \left( \frac{1}{2} + \frac{1}{e^{\beta \hbar w} - 1} \right), \quad C = \frac{\partial E}{\partial T}.$$

Also show that $C \approx k_B$ at high temperature and $C \to 0$ at low temperature.

## 3 Quantum Dice [30 pt]

Sethna (1.1).