Summary of Statistical Mechanics

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1 Thermodynamic Variables and Partition Function

The first and second laws of Thermodynamics are

$$dE = TdS - PdV. \tag{1}$$
$$\Delta S \geq 0. \tag{2}$$

All the thermodynamic variables $\langle E \rangle$, S, P, and their derivatives can be obtained from the partition function. First, define the partition function at a temperature T as

$$Z(T, V) = \sum_k e^{-\beta E_k}, \tag{3}$$

where

$$\beta = \frac{1}{k_B T}. \tag{4}$$

Next, we define the free energy

$$A(T, V) = -k_B T \log Z = \langle E \rangle - TS. \tag{5}$$

From the free energy and partition function we can obtain all other thermodynamic quantities such as the internal energy $\langle E \rangle$, entropy S, pressure P, specific heat C_V at a constant volume:

$$\langle E \rangle = -\frac{\partial \log Z}{\partial \beta}, \quad C_V = \frac{\partial \langle E \rangle}{\partial T}, \quad S = -\frac{\partial A}{\partial T}, \quad P = -\frac{\partial A}{\partial V}. \tag{6}$$

2 Quantum Statistical Mechanics

The average occupation number for "bosons" is

$$\langle n \rangle_{BE} = \frac{1}{e^{\beta(\epsilon - \mu)} - 1}. \tag{7}$$

The average occupation number for "Fermions" is

$$\langle n \rangle_{FD} = \frac{1}{e^{\beta(\epsilon - \mu)} + 1}. \tag{8}$$